# schubert calculus

**schubert calculus** is a fascinating and intricate area of mathematics that intersects algebraic geometry, combinatorics, and representation theory. This field primarily deals with the study of Schubert classes in the cohomology of Grassmannians and their applications in enumerative geometry. Schubert calculus provides powerful tools for solving problems related to counting geometric objects, understanding intersection theory, and exploring the structure of various mathematical spaces. In this article, we will delve into the fundamental concepts of Schubert calculus, its historical development, key applications, and the interplay between Schubert calculus and other mathematical domains. Furthermore, we will explore various methods and techniques used in this area, providing a comprehensive overview suitable for both beginners and advanced learners.



## Introduction to Schubert Calculus

Schubert calculus is named after Hermann Schubert, who formulated the initial ideas surrounding this mathematical framework in the late 19th century. At its core, Schubert calculus focuses on the study of Schubert varieties, which are geometric objects associated with the Grassmannian, a space that parametrizes linear subspaces of a given dimension in a vector space. The fundamental question in Schubert calculus often revolves around counting the number of intersections of these varieties under certain conditions.

The central objects of study in Schubert calculus are Schubert classes, which are cohomology classes that arise from Schubert varieties. These classes can be used to formulate problems in enumerative geometry, such as counting the number of lines intersecting a given number of surfaces in projective space. The beauty of Schubert calculus lies in its ability to provide explicit formulas and combinatorial interpretations for these enumerative problems.

# Historical Background

The origins of Schubert calculus trace back to the work of Hermann Schubert in the 1870s, where he introduced the concept of enumerating intersections of subspaces. Schubert's work laid the groundwork for the development of intersection theory and the study of enumerative geometry within algebraic geometry.

In the early 20th century, mathematicians like David Hilbert and others expanded upon Schubert's ideas, further formalizing the techniques used to count intersections and develop a more rigorous framework. The introduction of cohomology theory in the 1940s by mathematicians such as Henri Poincaré and Alexander Grothendieck provided the tools necessary to analyze Schubert classes in a more systematic manner.

Over the decades, Schubert calculus has evolved into a rich field of study, attracting interest from various mathematical disciplines, including algebraic geometry, combinatorial mathematics, and mathematical physics. The interplay between these areas has led to significant advancements in the understanding of complex geometric structures.

# Fundamental Concepts

To grasp the essentials of Schubert calculus, one must first understand several key concepts, including Grassmannians, Schubert varieties, and Schubert classes.

## Grassmannians

The Grassmannian, denoted as Gr(k, n), is the space of all k-dimensional linear subspaces of an n-dimensional vector space. It provides a natural setting for studying problems in linear algebra and geometry. For example, the Grassmannian Gr(1, n) corresponds to the projective space of lines in n-dimensional space.

## Schubert Varieties

Schubert varieties are specific subvarieties of the Grassmannian that correspond to particular conditions on the position of subspaces. Each Schubert variety can be associated with a partition, which dictates the intersections of these subspaces with respect to a chosen flag of subspaces.

## Schubert Classes

Schubert classes are cohomology classes represented by Schubert varieties. They are crucial in the formulation of intersection problems, allowing mathematicians to compute intersection numbers using algebraic methods. The product of Schubert classes corresponds to the intersection of their respective varieties, revealing deep combinatorial relationships.

# Applications of Schubert Calculus

Schubert calculus has numerous applications across various fields of mathematics and beyond. Some of the most prominent applications include:

- **Enumerative Geometry:** One of the primary applications of Schubert calculus is in enumerative geometry, where it provides tools to count geometric configurations, such as lines, planes, and higher-dimensional subspaces intersecting in specified ways.
- **Intersection Theory:** Schubert calculus plays a significant role in intersection theory, helping compute intersection numbers and understand the geometric properties of varieties.
- **Representation Theory:** The connections between Schubert calculus and representation theory have led to new insights in the study of symmetric functions and their applications in combinatorial enumeration.
- **Mathematical Physics:** Schubert calculus has found applications in mathematical physics, particularly in areas like string theory and the study of moduli spaces.

# Techniques and Methods

There are several techniques widely used in Schubert calculus, which facilitate the computation of intersection numbers and the study of Schubert classes.

## Intersection Numbers

Intersection numbers can be computed using various methods, including the use of the Schubert polynomial, which encodes the combinatorial structure of intersections. These polynomials allow for explicit calculations of intersection numbers via their coefficients.

## Combinatorial Techniques

Combinatorial techniques such as Young tableaux and the use of symmetric functions are instrumental in computing intersection numbers. These methods provide a visual and combinatorial approach, making the computations more accessible and understandable.

## Geometric Techniques

Geometric techniques involve studying the properties of Schubert varieties directly through their definitions and geometric configurations. This approach often leads to insights about the structure and relationships between different varieties.

# Interconnections with Other Areas of Mathematics

Schubert calculus is not only significant in its own right but also intersects with various other mathematical domains. Its connections are particularly notable in the following areas:

## Algebraic Geometry

Schubert calculus is rooted in algebraic geometry, particularly concerning the study of projective varieties and their cohomology. The methods developed within Schubert calculus have influenced the broader field of algebraic geometry significantly.

## Combinatorics

The combinatorial aspects of Schubert calculus have fostered a rich interaction between the two fields, leading to the development of new combinatorial identities and results related to symmetric functions and partitions.

## Topology

Topology, particularly algebraic topology, is closely linked to Schubert calculus through the study of cohomology rings and characteristic classes. This relationship enhances the understanding of the topology of various mathematical spaces.

# Future Directions and Research Trends

The field of Schubert calculus continues to evolve, with ongoing research exploring new applications and deeper theoretical insights. Current trends include:

- **Connection to Mathematics of Moduli Spaces:** The exploration of how Schubert calculus can be applied to the study of moduli spaces is an active area of research.
- **Computational Techniques:** Advances in computational methods are enabling researchers to tackle more complex problems and verify conjectures in Schubert calculus.
- **Interdisciplinary Applications:** The application of Schubert calculus techniques in fields like algebraic topology, mathematical physics, and combinatorial optimization is expanding, opening new avenues for exploration.

# Conclusion

Schubert calculus stands as a vital and dynamic area of mathematics, characterized by its rich history, fundamental concepts, and wide-ranging applications. As an intersection of various mathematical disciplines, it offers profound insights into the nature of geometric objects and their relationships. The techniques and methods developed within this field continue to impact both theoretical and applied mathematics, paving the way for future discoveries and advancements. With ongoing research and exploration, Schubert calculus remains a vibrant area of study, promising exciting developments in the years to come.

## Q: What is Schubert calculus used for?

A: Schubert calculus is primarily used in enumerative geometry to count geometric configurations and understand the intersections of subspaces. It has applications in algebraic geometry, representation theory, and mathematical physics.

## Q: Who is Hermann Schubert?

A: Hermann Schubert was a German mathematician in the 19th century who introduced the foundational concepts of Schubert calculus, particularly in the study of intersections of subspaces and enumerative geometry.

## Q: What are Schubert classes?

A: Schubert classes are cohomology classes associated with Schubert varieties in the Grassmannian. They are used to compute intersection numbers and provide insights into the geometric properties of varieties.

## Q: How does Schubert calculus relate to combinatorics?

A: Schubert calculus has strong combinatorial aspects, particularly in the use of tools like Young tableaux and symmetric functions, which help in calculating intersection numbers and understanding the combinatorial structures involved.

## Q: What are Grassmannians?

A: Grassmannians are spaces that parametrize all k-dimensional linear subspaces of an n-dimensional vector space. They serve as the foundational setting for studying Schubert varieties and classes in Schubert calculus.

## Q: Can Schubert calculus be applied in mathematical physics?

A: Yes, Schubert calculus has applications in mathematical physics, particularly in areas like string theory and the study of moduli spaces, demonstrating its interdisciplinary relevance.

## Q: What are intersection numbers in Schubert calculus?

A: Intersection numbers in Schubert calculus are numerical values representing the number of points at which various subspaces intersect, computed using methods like Schubert polynomials and combinatorial techniques.

## Q: What future research trends are emerging in Schubert calculus?

A: Future research trends in Schubert calculus include exploring connections to moduli spaces, advancing computational techniques, and expanding interdisciplinary applications across various fields of mathematics and physics.

# [Schubert Calculus]()



**schubert calculus: Schubert Calculus and Its Applications in Combinatorics and Representation Theory** Jianxun Hu, Changzheng Li, Leonardo C. Mihalcea, 2020-10-24 This book gathers research papers and surveys on the latest advances in Schubert Calculus, presented at the International Festival in Schubert Calculus, held in Guangzhou, China on November 6–10, 2017. With roots in enumerative geometry and Hilbert's 15th problem, modern Schubert Calculus studies classical and quantum intersection rings on spaces with symmetries, such as flag manifolds. The presence of symmetries leads to particularly rich structures, and it connects Schubert Calculus to many branches of mathematics, including algebraic geometry, combinatorics, representation theory, and theoretical physics. For instance, the study of the quantum cohomology ring of a Grassmann manifold combines all these areas in an organic way. The book is useful for researchers and graduate students interested in Schubert Calculus, and more generally in the study of flag manifolds in relation to algebraic geometry, combinatorics, representation theory and mathematical physics.

**schubert calculus: k-Schur Functions and Affine Schubert Calculus** Thomas Lam, Luc Lapointe, Jennifer Morse, Anne Schilling, Mark Shimozono, Mike Zabrocki, 2014-06-05 This book gives an introduction to the very active field of combinatorics of affine Schubert calculus, explains the current state of the art, and states the current open problems. Affine Schubert calculus lies at the crossroads of combinatorics, geometry, and representation theory. Its modern development is motivated by two seemingly unrelated directions. One is the introduction of k-Schur functions in the study of Macdonald polynomial positivity, a mostly combinatorial branch of symmetric function theory. The other direction is the study of the Schubert bases of the (co)homology of the affine Grassmannian, an algebro-topological formulation of a problem in enumerative geometry. This is the first introductory text on this subject. It contains many examples in Sage, a free open source general purpose mathematical software system, to entice the reader to investigate the open problems. This book is written for advanced undergraduate and graduate students, as well as researchers, who want to become familiar with this fascinating new field.

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**schubert calculus: Computations in Algebraic Geometry with Macaulay 2** David Eisenbud, Daniel R. Grayson, Mike Stillman, Bernd Sturmfels, 2013-03-14 Systems of polynomial equations arise throughout mathematics, science, and engineering. Algebraic geometry provides powerful theoretical techniques for studying the qualitative and quantitative features of their solution sets. Re cently developed algorithms have made theoretical aspects of the subject accessible to a broad range of mathematicians and scientists. The algorith mic approach to the subject has two principal aims: developing new tools for research within mathematics, and providing new tools for modeling and solv ing problems that arise in the sciences and engineering. A healthy synergy emerges, as new theorems yield new algorithms and emerging applications lead to new theoretical questions. This book presents algorithmic tools for algebraic geometry and experi mental applications of them. It also introduces a software system in which the tools have been implemented and with which the experiments can be carried out. Macaulay 2 is a computer algebra system devoted to supporting research in algebraic geometry, commutative algebra, and their applications. The reader of this book will encounter Macaulay 2 in the context of concrete applications and practical computations in algebraic geometry. The expositions of the algorithmic tools presented here are designed to serve as a useful guide for those wishing to bring such tools to bear on their own problems. A wide range of mathematical scientists should find these expositions valuable. This includes both the users of other programs similar to Macaulay 2 (for example, Singular and CoCoA) and those who are not interested in explicit machine computations at all.

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**schubert calculus: The Honors Class** Ben Yandell, 2001-12-12 This eminently readable book focuses on the people of mathematics and draws the reader into their fascinating world. In a monumental address, given to the International Congress of Mathematicians in Paris in 1900, David Hilbert, perhaps the most respected mathematician of his time, developed a blueprint for mathematical research in the new century.

**schubert calculus: Reality and Computation in Schubert Calculus** Nickolas Jason Hein, 2013 The Mukhin-Tarasov-Varchenko Theorem (previously the Shapiro Conjecture) asserts that a Schubert problem has all solutions distinct and real if the Schubert varieties involved osculate a rational normal curve at real points. When conjectured, it sparked interest in real osculating Schubert calculus, and computations played a large role in developing the surrounding theory. Our purpose is to uncover generalizations of the Mukhin-Tarasov-Varchenko Theorem, proving them when possible. We also improve the state of the art of computationally solving Schubert problems, allowing us to more effectively study ill-understood phenomena in Schubert calculus. We use supercomputers to methodically solve real osculating instances of Schubert problems. By studying over 300 million instances of over 700 Schubert problems, we amass data significant enough to reveal generalizations of the Mukhin-Tarasov- Varchenko Theorem and compelling enough to support our conjectures. Combining algebraic geometry and combinatorics, we prove some of these conjectures. To improve the efficiency of solving Schubert problems, we reformulate an instance of a Schubert problem as the solution set to a square system of equations in a higher- dimensional space. During our investigation, we found the number of real solutions to an instance of a symmetrically defined Schubert problem is congruent modulo four to the number of complex solutions. We proved this congruence, giving a generalization of the Mukhin-Tarasov-Varchenko Theorem and a new invariant in enumerative real algebraic geometry. We also discovered a family of Schubert problems whose number of real solutions to a real osculating instance has a lower bound depending only on the number of defining flags with real osculation points. We conclude that our method of computational investigation is effective for uncovering phenomena in enumerative real algebraic geometry. Furthermore, we point out that our square formulation for instances of Schubert problems may facilitate future experimentation by allowing one to solve instances using certifiable numerical methods in lieu of more computationally complex symbolic methods. Additionally, the methods we use for proving the congruence modulo four and for producing an unexpected square system of equations are both quite general, and they may be of use in future projects. The electronic version of this dissertation is accessible from http://hdl.handle.net/1969.1/151084

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Topology Javier Fernández de Bobadilla, Tamás László, András Stipsicz, 2021-05-27 The book is a collection of surveys and original research articles concentrating on new perspectives and research directions at the crossroads of algebraic geometry, topology, and singularity theory. The papers, written by leading researchers working on various topics of the above fields, are the outcome of the "Némethi60: Geometry and Topology of Singularities" conference held at the Alfréd Rényi Institute of Mathematics in Budapest, from May 27 to 31, 2019. Both the conference and this resulting volume are in honor of Professor András Némethi, on the occasion of his 60th birthday, whose work plays a decisive and influential role in the interactions between the above fields. The book should serve as a valuable resource for graduate students and researchers to deepen the new perspectives, methods, and connections between geometry and topology regarding singularities.

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